Interval Cognitive Maps

Y.G. Petalas[*1], **E.I. Papageorgiou**[2], **K.E. Parsopoulos**[1], **P.P. Groumpos**[2] and **M.N. Vrahatis**[1]

[1] Computational Intelligence Laboratory (CI Lab), Department of Mathematics, University of Patras Artificial Intelligence Research Center (UPAIRC), University of Patras, GR-26110 Patras, Hellas

[2] Department of Electrical Engineering, Lab. of Automation and Robotics, University of Patras, GR–26500 Patras, Hellas

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Fuzzy Cognitive Maps (FCMs) model the behavior of a system by using concepts and their relationships. They contain nodes–concepts and weighted edges that connect the nodes and represent the cause and effect relationships among them. Both nodes and edges are fuzzy sets and are bounded in ranges provided by the experts for the problem under consideration. Membership functions are firstly determined for each fuzzy set, and fuzzification and defuzzification processes are applied for the initial crisp values. In this work, the Interval Cognitive Maps (ICMs) are proposed and the associated learning task is accomplished by the optimization of an objective function. ICMs share the same structure with FCMs but their main difference is that the concepts and the weights of FCMs are not fuzzy sets but rather interval numbers.

1 Introduction

Fuzzy Cognitive Maps (FCMs) constitute a promising modeling methodology that provides flexibility on the simulated system's design, modeling and control. They were introduced by Kosko for the representation of causal relationships among concepts, as well as, for the analysis of inference patterns [1, 2]. Up–to–date FCMs have been applied in various scientific fields, including bioinformatics, manufacturing, organization behavior, political science, and decision making. Although FCMs constitute a promising modeling methodology, they have some deficiencies regarding the robustness of their inference mechanism and their ability to adapt the experts' knowledge through optimization and learning [1, 2]. FCMs are described by a graph with the nodes representing concepts and the arcs between the concepts denoting the relationship between them. The successful operation of FCMs relies on the proper determination of their weights.

Concepts and weights are fuzzy sets and their values are restricted within a range of values which are given by the experts for each problem domain. One could also treat the concepts and the weights, as interval numbers that lie within the range provided by the experts, rather than as fuzzy sets. This new model is called Interval Cognitive Maps (ICMs).

Interval analysis is a deterministic way of representing uncertainty in values by replacing a number with a range of values. Interval analysis was introduced to deal with numerical errors which occurred in mathematical computations performed on digital computers. The result of an ordinary computation (non–interval) is a single number, a point on the real number line, which lies at some unknown distance from the true answer. An interval computation yields a pair of numbers, an upper and a lower bound which are guaranteed to enclose the exact answer. Until now, interval analysis has been used in many branches of mathematics, including numerical analysis, probability and logic.

The remaining paper is organized as follows: Section 2 describes FCMs and the relation with interval analysis giving rise to ICMs, while in Section 3 experimental results are given from the application of the proposed methodology. In Section 4, the paper ends with conclusions.

* Corresponding author: e-mail: petalas@math.upatras.gr, Phone: +30 2610 997378, Fax: +30 2610 992965

2 Fuzzy Cognitive Maps and Interval Cognitive Maps

FCMs combine properties of fuzzy logic and neural networks. An FCM models the behavior of a system using concepts, C_i, $i = 1, \ldots, N$, that represent the states, variables, or characteristics of the system. The system is then represented by a fuzzy signed directed graph with feedback. It contains nodes–concepts and weighted edges that connect the nodes and represent the cause and effect relations among them. The values, A_i, of the concepts lie within $[0, 1]$ and they are susceptible to change over time. The weights, W_{ij}, of the edges assume values in $[-1, 1]$, and represent the extent of the impact of the interconnected concepts on each other. The design of an FCM is a process that heavily relies on the input from a group of experts and results in an initial weight matrix, $W^{\text{initial}} = [W_{ij}]$, with $W_{ii} = 0$, $i, j = 1, \ldots, N$. After the determination of its structure, the FCM converges to a steady state by applying the rule,

$$A_i(t+1) = f\left(A_i(t) + \sum_{\substack{j=1 \\ j\neq i}}^{n} W_{ji} A_j(t)\right),$$

with arbitrary initial values of A_i [2], where t stands for the time index. The function f is a threshold function, usually the logistic function.

The main goal of learning in FCMs is to determine the values of the weights of the FCM, that produce the desired behavior of the system. The desired behavior of the system is characterized by values of the output concepts that lie within the prespecified bounds, determined by the experts.

Recently, Muata et al. [3] used the interval pairwise comparison techniques to provide effective means to generate consistent estimates of the magnitude of each causal relationship. The presented learning evaluation procedure generates a consistent set of magnitudes of the causal relationships of an FCM, and provides a flexible and robust approach for reaching closure, even in the presence of ambiguity, and supports convergence to point estimates.

In this paper, we present ICMs. ICMs are very similar with FCMs, they have the same structure and the same rule for evolution. The key difference between ICMs and FCMs lies in the representation of concepts and weights. In FCMs, concepts and weights are fuzzy sets. So they must be associated with a membership function and also a fuzzification and defuzzification process must take place. On the contrary, in ICMs, concepts and weights are treated as interval numbers. The upper and lower bounds of each interval coincides with the ranges given initially by the experts for the concepts and the weights. Thus, ICM is a simpler model since neither the fuzzification and defuzzification process, nor the selection of the proper membership function for each fuzzy set, is required.

There are several types of errors in mathematical computations. Data may be uncertain, rounding errors generally occur, approximations are made, etc. The purpose of interval analysis is to provide upper and lower bounds of all such errors on a computed quantity. By an interval we mean a closed bounded set of 'real' numbers:

$$[a, b] = \{x : a < x < b\}.$$

We can also regard an interval as a number represented by the ordered pair of its endpoints a and b. Thus intervals have a dual nature, representing a set of real numbers by a new kind of number. This has led to the development of both interval arithmetic and interval analysis [4, 5, 6, 7].

Interval analysis has been applied in logic, probability, computation of the range of functions (global optimization), and validation methods for the solution of equations. These have direct and indirect relationships to fuzzy set theory. Interval methods are useful whenever we have to deal with uncertainties, which can be rigorously bounded. Fuzzy sets, rough sets and probability calculus can perform similar tasks, yet only interval methods are able to prove, with mathematical rigor, the nonexistence of desired solution(s).

Our learning approach specifies and generates a consistent set of weights representing causal relationships of an ICM. As in FCMs learning [8, 9], for the problem under consideration, an objective function is created and its global minimization with respect to the weights of the ICM leads the ICM to a desired state. Thus, the task of ICMs learning reduces to a global optimization task with interval numbers [5].

3 Experimental Results

We have applied the proposed methodology on a medical decision problem for radiation therapy [9]. The following ranges for the weight values were suggested by three experts: $0.3 \leqslant w_{16} \leqslant 0.5$, $0.2 \leqslant w_{21} \leqslant 0.4$, $0.5 \leqslant w_{26} \leqslant 0.7$, $-0.4 \leqslant w_{32} \leqslant -0.2$, $-0.4 \leqslant w_{36} \leqslant -0.1$, $-0.5 \leqslant w_{45} \leqslant -0.2$, $-0.5 \leqslant w_{46} \leqslant -0.2$, $-0.6 \leqslant w_{54} \leqslant -0.1$, $0.5 \leqslant w_{56} \leqslant 0.8$, $0.2 \leqslant w_{61} \leqslant 0.4$, $0.6 \leqslant w_{62} \leqslant 0.9$, $0.5 \leqslant w_{65} \leqslant 0.9$.

This information from the experts has been exploited by incorporating these ranges as constraints on the parameter vector W, in the experiments conducted. These ranges define the intervals for each weight.

The objective function that was created for this medical problem is the following:

$$F(W) = -\textit{FD(W)} - D(W), \tag{1}$$

where *FD(W)* and $D(W)$ are the values of the final dose and the dose [9], respectively, that correspond to the weight matrix W. The minus signs are used to transform the maximization problem to its equivalent minimization problem.

The global optimization package GlobSol, was used for the global optimization process. GlobSol is a Fortran 90 well-tested and self-contained package. It solves constrained and unconstrained global optimization problems. Among others, it has a separate program for solving a square algebraic system of equations, utility programs for interval and point evaluation and subroutine / module libraries for interval arithmetic and automatic differentiation [10].

The final solution was:

$$W^* = \begin{pmatrix} 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.5 \\ 0.4 & 0.0 & 0.0 & 0.0 & 0.0 & 0.7 \\ 0.0 & -0.2 & 0.0 & 0.0 & 0.0 & -0.1 \\ 0.0 & 0.0 & 0.0 & 0.0 & -0.2 & -0.2 \\ 0.0 & 0.0 & 0.0 & -0.6 & 0.0 & 0.8 \\ 0.4 & 0.9 & 0.0 & 0.0 & 0.9 & 0.0 \end{pmatrix},$$

which fulfills the constraints of the weights posed by the experts and is also the optimal solution [9].

The proposed procedure provides for exploration of the decision domain, facilitating the expert's understanding of the realities of a given decision making situation where there is ambiguity, and aiding the expert to reach closure.

The proposed approach using interval numbers for the input and output concepts, as well as for the weights, thus yielding ICMs, appears to be more efficient and faster than the existing FCMs learning approaches due to the fact that no membership functions are determined and there is no need for fuzzification and defuzzification processes.

4 Conclusions

A new model, called ICM, has been proposed. ICMs have the same structure and the same rule for evolution as FCMs, but their structural elements, concepts and relationships among them, that were fuzzy sets in FCMs, are now interval numbers. Learning in ICMs relies on the optimization of an objective function constructed for the problem under consideration. In this work, ICM was successfully applied to a medical application concerning radiation therapy. In a future work, we intent to investigate the effectiveness of the proposed methodology to other "real - life" applications, and compare its performance with alternative methods.

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