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ABSTRACTS

Department of Mathematics,
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Michael N. Vrahatis
Department of Mathematics, University of Patras, GR-26110 Patras, Greece
e-mail: vrahatis@math.upatras.gr

Extensions of the Knaster-Kuratowski-Mazurkiewicz covering principle

Mathematics Subject Classification (MSC): Primary 55M20, 54H25; Secondary 47H10, 65H10, 91A44, 91B50

Abstract. The important and pioneering *"KKM covering principle"* (1929) [4] for an n-dimensional simplex in $\mathbb{R}^n$, due to **Bronisław Knaster**, **Casimir (Kazimierz) Kuratowski** and **Stefan Mazurkiewicz**, also known as "Knaster– Kuratowski–Mazurkiewicz lemma" ("KKM lemma" for short), is presented and analyzed.

It is well known that many significant theorems including, among others, the famous "Brouwer's fixed point theorem" (1912) [2] due to **Luitzen Egbertus Jan Brouwer**, can be proved using the KKM covering principle. Furthermore, three pioneering classical results, namely:

(a) the "Brouwer fixed point theorem" (1912) [2],

(b) the "Sperner lemma" (1928) due to **Emanuel Sperner** [6], and

(c) the "KKM lemma" (1929) [4],

are mutually equivalent in the sense that each one can be deduced from another. In addition, the KKM lemma has a plethora of applications in various fields of pure and applied mathematics.

Extensions and applications of the KKM lemma are presented and analyzed. Specifically, a proof of a generalization of the "Bolzano's theorem" also called "intermediate value theorem", given independently by **Bernard Bolzano** (1817) [1] and **Augustin-Louis Cauchy** (1821) [3], is presented [7]. This proof is based on the KKM covering principle. In addition, among others, in the field of mathematical economics, the very important and pioneering extension of the KKM lemma due to **Lloyd Stowell Shapley** [Nobel Laureate in Economic Sciences in 2012] customarily called the

"Knaster-Kuratowski-Mazurkiewicz-Shapley theorem" (KKMS theorem for short) (1973) [5], is presented. It is worth noting that, the *"KKMS covering principle"* constitutes the basis for the proof of many theorems that are related to the existence of solutions in "Game Theory" and in the "General Equilibrium Theory" of economic analysis.

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