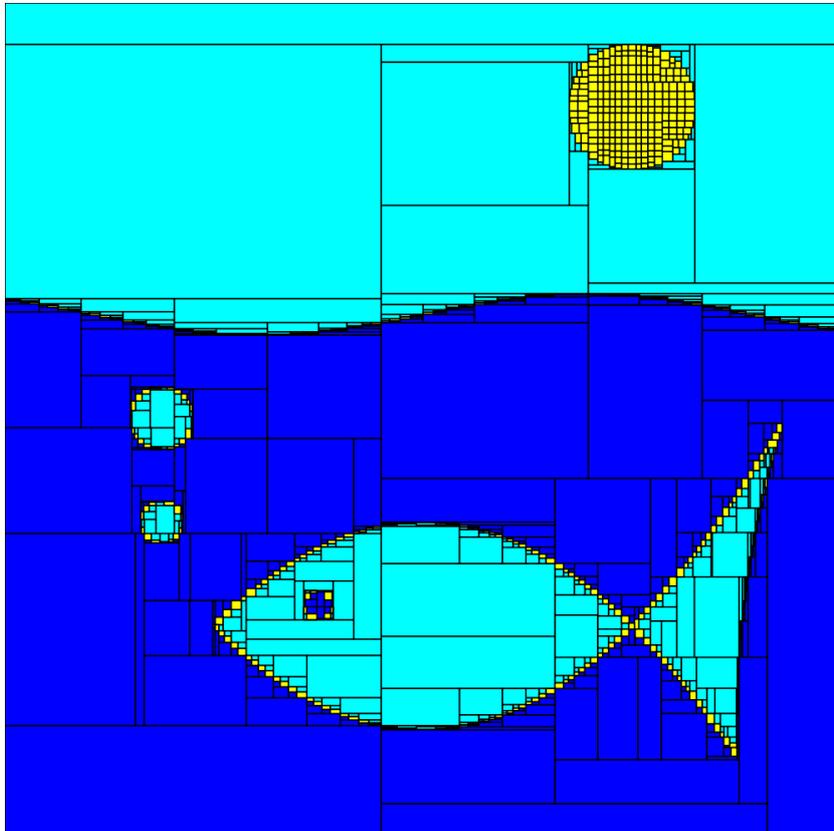

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BOOK OF ABSTRACTS

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Interval Methods for Resolving Neural Computation Issues

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Introduction

Effective handling of uncertainty constitutes an important issue when dealing with systems built on concepts and methods from the areas of Computational Intelligence. Uncertainty in such systems appears in various forms both when modeling a real process, as well as, during deployment and exploitation of a system, and typically, it is related to the input data and to model parameters. A multitude of research efforts are reported in the literature, concerning its quantification and effective handling. The majority of these efforts adopt concepts and methods arising from areas such as, probabilities and stochastic processes, Bayesian theory, fuzzy logic, mathematical theory of evidence, rough sets, etc. However, these approaches have no means to handle numerical errors, while at the same time it is questionable whether they can effectively tackle uncertainty without specific knowledge of the underlying process, such as the knowledge of an expert, or in the absence of known probability distributions, etc.

Interval arithmetic was introduced as a means to perform numerical computations with guaranteed accuracy and bounding the ranges of the quantities, used in the computations. Nowadays, interval analysis offers a whole toolbox for providing reliable solutions to several problems, especially, when they concern processes for which the proposed model can be cast in some closed-form or analytic expression.

The study and the use of interval analysis has attracted the interest of researchers in the area of computational intelligence, and more specifically, in the field of neural networks. Various trends are reported in the literature

regarding the use of interval analysis approaches in artificial neural networks. Most of them focus on endowing neural networks with the capability to process uncertain data expressed in the form of intervals, while the “maximalist approach”, in this direction, concerns the conception and the implementation of interval neural networks i.e., neural architectures which not only are capable to process interval valued data but they, also, dispose some suitable training mechanism based on interval optimization techniques, [1]. Other notable research efforts dealing with the integration of the ability to process interval data into classical neural networks, range from, suitably modifying neural architectures, to proposing interval-like versions of the classical training algorithms such as gradient descent, [2,3].

Recent Research Results

Recently, aiming at these objectives has been reconsidered from a rather different point of view. In contrast to maximalist approaches or to some interval-mimicking techniques and algorithms, the work of Adam et al. [4,5,6] focused on using interval methods in order to resolve specific neural computation issues for which there exists some appropriate transcription in terms of intervals. In order to derive intervals for the values of the critical parameters of the networks, we had to revisit fundamental neural network concepts and well known approaches and identify the level of effective interventions. On the other hand, in order for these interventions to be successful we had to study and define the necessary theoretical tools for providing support to the effective application of the interval methods. The issues, presented hereafter, were identified for potential improvement or resolution.

Effective weight initialization of a neural network, [4]: The problem of determining good initial conditions for a local search algorithm used to train a multi-layer perceptron (MLP) was studied. For each node in the hidden layer synaptic weights are considered to be located within the bounds of some unknown intervals. These intervals together with the intervals of the values of the signals, input to a node, form an interval linear system corresponding to a linear interval tolerance problem. A number of theoretical results are proved and a new algorithm is proposed for solving this problem and, hence, for defining effective intervals for the initial weights. The proposed approach inherently includes some of the major concepts involved in neural network weight initialization, such as: the number of inputs to a node in the first hidden layer, the statistical information of the input data, effective

positioning of the hyper-planes in the pattern space and full utilization of the dynamic range of the activation function. The proposed method is tested on a number of well known benchmarks for MLPs trained with some well known back-propagation algorithms and the experimental results obtained are compared against the results of a number of well known and established weight initialization methods.

Definition of the area in the weight space of an MLP where a global minimizer of the network’s output error function is guaranteed to be located, [5]: Using global optimization techniques for neural network training has been an important issue in the field of neural computation as these techniques succeed to find a global minimizer of the network’s error function while avoiding the problems related to local search. However, a major problem that still remains to be solved concerns the region where these methods will effectively search for some global optimizer. Given that the weight space of an MLP is unbounded, the current practice on this matter consists in, heuristically, defining a bounded region hoping that some global optimizer is contained in there. The approach elaborated in this research relies on interval analysis and defines guaranteed bounds of the region in the search space where some global search algorithm should operate when training an MLP. These bounds, depend on the machine precision set for the solution of the problem and the term guaranteed denotes that the region determined surely encloses weight vectors that are global minimizers of the neural network’s error function. Generally, the solution set of this bounding problem of an MLP is non-convex. However, the theoretical results elaborated helped deriving a box which is a convex set. This box is an outer approximation of the algebraic solutions to the interval equations resulting from the functions implemented by the network nodes.

Reliable estimation of an MLP’s domain of validity, [6]: The quality of training of a neural network is represented to a great extent by its domain of validity as it helps to assess the network’s ability to cope with a given problem. A number of research efforts can be found in the literature on this matter aiming to provide as accurate estimations of the domain of validity as possible. Given that the dependence of a neural network output on the pattern data is a nonlinear function, in this research, we consider that derivation of the area in the input space, effectively taken into account by the neural network function, can be addressed as a nonlinear parameter estimation problem. Hence, this problem can be tackled by SIVIA, the approach

originally introduced by Jaulin and Walter [7]. The use of interval computation, obviously, guarantees the reliability of the results in terms of accurately detecting the domain of validity. The proposed method was experimentally tested on a number of problems and the results obtained proved to be very promising for obtaining reliable conclusions on aspects of the neural network such as its ability to generalize well and its ability to be explicative.

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