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Quantum deformed problem of electrons in the Dirac theory

In memory of professor Gregor Tsagas

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Abstract

In the present paper we investigate the deformed problem of the Dirac electrons in the non – commutative geometry and in the Lie – admissible formulation of the Quantum Gravity. The time and momentum deformations are introduced through the Caldirola – Montaldi (C.M.) model as well as the Small – Distance – Derivative (S.D.D.) model. The above models are special cases of the Lie – admissible theory. The results are based on the theory developed by Gonzalez - Diaz who used the (S.D.D.) model to construct a modified Lie – admissible Wheeler – De Witt equation. The interpretation of this equation is that the universe has a non zero total energy where values coincide with the corresponding values of a harmonic oscillator with Planck mass M^*. It is an open system which interacts with some sort of "exterior" World and it is created by a kind of physical reality. Also the group velocity, after applying the two models, to the Dirac theory, leads to a velocity greater of c and satisfies the Santilli's hypothesis.

1 Introduction

Recently Jannussis *et al* [1] in the paper non – commutative geometry and applications in physics by using the Bopp – Kubo [2, 3] formulation and the Weyl – Wigner calculus of quantum mechanics in phase space, which is found to arise from the fact the Moyal product of function on phase $f(\phi) * g(\phi)$ can be rewritten equivalently as the product of functions defined on the extended phase space $f(\Phi) * g(\phi)$ [4].

From the non – commutative geometry point of view the phase space formulation of quantum mechanics is an example of the theory with non – commutative geometry. According to ref [4] the non – commutative algebra A is the algebra of functions on phase space endowed with a Moyal product. The right and left A – modules are interchanged by the modular conjugation J, so that the space of functions $\varepsilon = A \otimes A/J$, and there is a kind of chiral symmetry due to the non – commutativity.

Recently studies of non – commutative phase space are much in the spirit of modern non – commutative geometry [5, 6, 7, 8]. The applications and the results of non – commutative geometry to many branches of physics [9, 10, 11, 12] even in the string theories [13, 14] are of great interest. According to ref [12], non – commutativity in string theory and ordinary gauge theories are equivalent. In ref [1] we have used the method of Bopp – Kubo for different forms of the Hamilton operator.

$$\mathcal{H}(\vec{q},\vec{p}) = \frac{\vec{p}^2}{2m} + V(\vec{q}) \tag{1.1}$$

where $\vec{q}(q_1, q_2, q_3)$ and $\vec{p}(p_1, p_2, p_3)$ are the space and momentum coordinates with the commutation relations [1, 15, 16]

$$[p_1, p_2] = i\lambda, \qquad [q_1, q_2] = i\theta, \qquad [q_j, p_k] = i\delta_{jk}\hbar \tag{1.2}$$

$$[p_3, p_1] = 0, \qquad [p_3, p_2] = 0, \qquad [q_3, q_1] = 0, \qquad [q_3, q_2] = 0 \tag{1.3}$$

and λ, θ are real and positive numbers. The physical significance of the θ parameter, for which $\sqrt{\theta} = L_p$ (L_p is the Planck length), is referred in ref [9, 17, 18]. Also according to the above references, it is impossible to construct an apparatus which will measure length scales smaller than the Planck length. These effects exist even in flat space – time because of vacuum fluctuations on gravity. These fluctuations lead to a high energy or ($\sqrt{\lambda}$ – momentum) cut – off in flat space field theories by removing the divergence problem.

For $\theta = 0$ and $\lambda = B\hbar^2$, where B has the dimensions inverse surface, the length $L = \sqrt{1/B} = \sqrt{\frac{c\hbar}{eH}} \approx 10^{-5}cm$ is mainly used to experimental applications for the cyclotron resonances with the symmetric vector potential $\vec{A}(\vec{q}) = \frac{1}{2}|\vec{H} \times \vec{q}|$, where H is constant magnetic field along the z – axis. The Hamilton operator (1.1) in momentum non – commutative picture coincides exactly with the well – known Landau Hamilton operator i.e.

$$\mathcal{H}(\vec{q},\vec{p}) = -\frac{\hbar^2}{2m}\left(\vec{\nabla} - \frac{ie}{c\hbar}\vec{A}(\vec{q})\right)^2 + V(\vec{q}) \tag{1.4}$$

Also for the Dirac operator $\mathcal{H}_D$ we obtain the same result.

In section 2 we will study the case in relativistic non – commutative geometry according to the commutation relations (1.2),(1.3) for the free electron.

In section 3 we have used different models which are partial cases of the Santilli's [19] "Hadronic Mechanics". We obtain new results for the quantum gravity and the Lie – admissible structure of the universe.

Section 4 concludes the paper.

2 Dirac equation in non – commutative geometry

According to commutation relation (1.2),(1.3) the Dirac differential system in non – commutative geometry has the following form

$$\left[i\frac{\hbar}{c}\frac{\partial}{\partial t} + \frac{e}{c}V(q_1,q_2,q_3) + m_0c\right]\Psi_1 + \left[(p_1 - ip_2) - \frac{\lambda}{2}\left(\frac{\partial}{\partial p_1} - i\frac{\partial}{\partial p_2}\right)\right]\Psi_4 + p_3\Psi_3 = 0$$

$$\left[i\frac{\hbar}{c}\frac{\partial}{\partial t} + \frac{e}{c}V(q_1,q_2,q_3) + m_0c\right]\Psi_2 + \left[(p_1 + ip_2) + \frac{\lambda}{2}\left(\frac{\partial}{\partial p_1} + i\frac{\partial}{\partial p_2}\right)\right]\Psi_3 - p_3\Psi_4 = 0$$

$$
\begin{aligned}
&\left[i\frac{\hbar}{c}\frac{\partial}{\partial t}+\frac{e}{c}V(q_1,q_2,q_3)-m_0c\right]\Psi_3+\left[(p_1-ip_2)-\frac{\lambda}{2}\left(\frac{\partial}{\partial p_1}-i\frac{\partial}{\partial p_2}\right)\right]\Psi_2+p_3\Psi_1=0\\
&\left[i\frac{\hbar}{c}\frac{\partial}{\partial t}+\frac{e}{c}V(q_1,q_2,q_3)-m_0c\right]\Psi_4+\left[(p_1+ip_2)+\frac{\lambda}{2}\left(\frac{\partial}{\partial p_1}+i\frac{\partial}{\partial p_2}\right)\right]\Psi_1-p_3\Psi_2=0
\end{aligned}
\tag{2.1}
$$

where

$$
q_1\to q_1+i\frac{\theta}{2}\frac{\partial}{\partial q_2}=q_1-\frac{\theta}{2\hbar}p_2,\qquad q_2\to q_2-i\frac{\theta}{2}\frac{\partial}{\partial q_1}=q_2+\frac{\theta}{2\hbar}p_1,\qquad p_3=p_3 \tag{2.2}
$$

For the case $\theta=0$ and $\lambda=\hbar^2B=\frac{e\hbar H}{c}$ the system (2.1) coincide exactly with the Dirac electron in a uniform magnetic field with the vector potential $\vec{A}(\vec{q})=\frac{1}{2}|\vec{H}\times\vec{q}|$

In ref [1] we have studied the problem of free Dirac electron in λ – non – commutative geometry by the following system:

$$
\begin{aligned}
&E_+\Psi_1+\left[(p_1-ip_2)-\frac{\lambda}{2}\left(\frac{\partial}{\partial p_1}-i\frac{\partial}{\partial p_2}\right)\right]\Psi_4+p_3\Psi_3=0\\
&E_+\Psi_2+\left[(p_1+ip_2)+\frac{\lambda}{2}\left(\frac{\partial}{\partial p_1}+i\frac{\partial}{\partial p_2}\right)\right]\Psi_3-p_3\Psi_4=0\\
&E_-\Psi_3+\left[(p_1-ip_2)-\frac{\lambda}{2}\left(\frac{\partial}{\partial p_1}-i\frac{\partial}{\partial p_2}\right)\right]\Psi_2+p_3\Psi_1=0\\
&E_-\Psi_4+\left[(p_1+ip_2)+\frac{\lambda}{2}\left(\frac{\partial}{\partial p_1}+i\frac{\partial}{\partial p_2}\right)\right]\Psi_1-p_3\Psi_2=0
\end{aligned}
\tag{2.3}
$$

where

$$
E_\pm=\frac{E}{c}\pm m_0c \tag{2.4}
$$

By using the well – known "Schraubenfunktionen" [20] we obtain the solution of the above system with the eigenvalues

$$
E=\pm c\sqrt{m_0^2c^2+2\lambda n+\hbar^2k_3^2},\qquad p_3=\hbar k_3 \tag{2.5}
$$

We continue with the same mechanisms l times and we obtain the following eigenvalues:

$$
E=\pm c\sqrt{m_0^2c^2+2ln\lambda+\hbar^2k_3^2},\qquad p_3=\hbar k_3. \tag{2.6}
$$

Also the system (2.3), according to the relations $q_j=i\hbar\frac{\partial}{\partial p_j}\delta_{jk}$, can be written

$$
\begin{aligned}
&E_+\Psi_1+\left[(p_1-ip_2)+\frac{i\lambda}{2\hbar}(q_1-iq_2)\right]\Psi_4+p_3\Psi_3=0\\
&E_+\Psi_2+\left[(p_1+ip_2)-\frac{i\lambda}{2\hbar}(q_1+iq_2)\right]\Psi_3-p_3\Psi_4=0
\end{aligned}
\tag{2.7}
$$

$$E_-\Psi_3 + \left[(p_1 - ip_2) + \frac{i\lambda}{2\hbar}(q_1 - iq_2)\right]\Psi_2 + p_3\Psi_1 = 0$$
$$E_-\Psi_4 + \left[(p_1 + ip_2) - \frac{i\lambda}{2\hbar}(q_1 + iq_2)\right]\Psi_1 - p_3\Psi_2 = 0$$

For the case $\theta \neq 0$, i.e. θ – non – commutativity we use the relations

$$q_1 \rightarrow q_1 - \frac{\theta}{2\hbar}p_2, \qquad q_2 \rightarrow q_2 + \frac{\theta}{2\hbar}p_1. \tag{2.8}$$

The system (2.7) takes the form

$$\begin{aligned}
E_+\Psi_1 + \left[\left(1 + \frac{\lambda\theta}{4\hbar^2}\right)(p_1 - ip_2) + \frac{i\lambda}{2\hbar}(q_1 - iq_2)\right]\Psi_4 + p_3\Psi_3 = 0 \\
E_+\Psi_2 + \left[\left(1 + \frac{\lambda\theta}{4\hbar^2}\right)(p_1 + ip_2) - \frac{i\lambda}{2\hbar}(q_1 + iq_2)\right]\Psi_3 - p_3\Psi_4 = 0 \\
E_-\Psi_3 + \left[\left(1 + \frac{\lambda\theta}{4\hbar^2}\right)(p_1 - ip_2) + \frac{i\lambda}{2\hbar}(q_1 - iq_2)\right]\Psi_2 + p_3\Psi_1 = 0 \\
E_-\Psi_4 + \left[\left(1 + \frac{\lambda\theta}{4\hbar^2}\right)(p_1 + ip_2) - \frac{i\lambda}{2\hbar}(q_1 + iq_2)\right]\Psi_1 - p_3\Psi_2 = 0
\end{aligned} \tag{2.9}$$

By continuing the same mechanism l – times in the θ – non – commutativity, we obtain in the p – representation the system

$$\begin{aligned}
E_+\Psi_1 + \left[\left(1 + l\frac{\lambda\theta}{4\hbar^2}\right)(p_1 - ip_2) - \frac{\lambda}{2}\left(\frac{\partial}{\partial p_1} - i\frac{\partial}{\partial p_2}\right)\right]\Psi_4 + p_3\Psi_3 = 0 \\
E_+\Psi_2 + \left[\left(1 + l\frac{\lambda\theta}{4\hbar^2}\right)(p_1 + ip_2) + \frac{\lambda}{2}\left(\frac{\partial}{\partial p_1} + i\frac{\partial}{\partial p_2}\right)\right]\Psi_3 - p_3\Psi_4 = 0 \\
E_-\Psi_3 + \left[\left(1 + l\frac{\lambda\theta}{4\hbar^2}\right)(p_1 - ip_2) - \frac{\lambda}{2}\left(\frac{\partial}{\partial p_1} - i\frac{\partial}{\partial p_2}\right)\right]\Psi_2 + p_3\Psi_1 = 0 \\
E_-\Psi_4 + \left[\left(1 + l\frac{\lambda\theta}{4\hbar^2}\right)(p_1 + ip_2) + \frac{\lambda}{2}\left(\frac{\partial}{\partial p_1} + i\frac{\partial}{\partial p_2}\right)\right]\Psi_1 - p_3\Psi_2 = 0.
\end{aligned} \tag{2.10}$$

The above system for

$$P_1 = \left(1 + l\frac{\lambda\theta}{4\hbar^2}\right)p_1, \qquad P_2 = \left(1 + l\frac{\lambda\theta}{4\hbar^2}\right)p_2 \quad and \quad \lambda' = \lambda\left(1 + l\frac{\lambda\theta}{4\hbar^2}\right) \tag{2.11}$$

coincides exactly with the system (2.3) and the corresponding eigenvalues are given in the following form

$$E = \pm c\sqrt{m_0^2c^2 + 2\lambda' n + \hbar^2k_3^2} = \pm c\sqrt{m_0^2c^2 + 2\lambda\left(1 + l\frac{\lambda\theta}{4\hbar^2}\right)n + \hbar^2k_3^2} \tag{2.12}$$

From the two forms (2.6,2.12) of the energy eigenvalues with $\lambda = \hbar^2 B = \frac{e\hbar H}{c}$, we see that the constant magnetic field H is multiplied by the integer number l and according to ref [17, 18] there exists a cut – off of the momentum $\sqrt{l\lambda}$ and a maximum number $l = l_{\max}$ which correspond to a very strong magnetic field on the Max Planck's scale. Also we see that, beginning with a constant magnetic field H, by using the above results we obtain a new very strong magnetic field.

3 Dirac equation in (C.M.) and (S.D.D.) models.

According to ref. [21, 22] the new momentum and the time derivative operators in the Caldirola – Montaldi (C.M.) model [23] take the following oscillating forms

$$p \rightarrow \frac{\hbar}{L} \sin \frac{Lp}{\hbar}, \qquad \frac{d}{dt} \rightarrow \frac{1}{\tau} \sinh \tau \frac{d}{dt} \tag{3.1}$$

where L is the Caldirola length, p is the usual momentum operator and τ is the Caldifola "chronon". The Caldirola L represents the critical distance, which can cause the space interaction to begin. The Caldirola chronon represents the time of the interaction between two physical systems.

In the three dimensional relativistic case we use the new deformed four – momentum vector (P_0, P_1, P_2, P_3)

$$P_0 \rightarrow \hbar k \sinh \frac{p_0}{\hbar k} = \kappa \sinh \frac{p_0}{\kappa}, \qquad p_j \rightarrow \frac{\hbar}{L_j} \sin \frac{L_j p_j}{\hbar} \qquad j = 1, 2, 3 \tag{3.2}$$

where $\hbar k = \kappa$ represents a fundamental mass parameter according to Lukierski *et al* [24].

Also in the small – distance – derivative (S.D.D.) model, according to ref [21] the new four – momentum operators are given by

$$P_0 \rightarrow i \frac{\hbar}{c} \frac{\partial}{\partial t} \cos \tau \frac{\partial}{\partial t} \qquad P_j \rightarrow p_j \cos \frac{L_j p_j}{\hbar}, \qquad j = 1, 2, 3. \tag{3.3}$$

When there is an electromagnetic field present, the operators (3.2) take the form

$$P_j \rightarrow p_j \cos \frac{L_j p_j}{\hbar} - \frac{e}{c} A_j \qquad P_4 = i \frac{\hbar}{c} \frac{\partial}{\partial t} \cos \tau \frac{\partial}{\partial t} - \frac{e}{c} V \tag{3.4}$$

where A_j are the components of the vector potential and V is the scalar potential.

The deformed relativistic case without magnetic fields, according to ref [21] for the two models (C.M.) and (S.D.D.) yields

$$\omega_1 = \frac{1}{\tau_1} \arcsin \frac{\tau_1 E_1}{\hbar} \qquad (C.M. model) \tag{3.5}$$

$$and \qquad \hbar \omega_2 \cos \omega \tau_2 = E_2 \qquad (S.D.D.\ model) \tag{3.6}$$

where, τ_1 is the Caldirola time and τ_2 the time related to the S.D.D. model, with the following eigenvalues:

$$E_1 = \pm c \left(\frac{\hbar^2}{L_1^2} \sin^2 k_1 L_1 + \frac{\hbar^2}{L_2^2} \sin^2 k_2 L_2 + \frac{\hbar^2}{L_3} \sin^2 k_3 L_3 + m_0^2 c^2 \right)^{1/2} \tag{3.7}$$

$$E_2 = \pm c \left(\hbar^2 k_1^2 \cos^2 k_1 L_1 + \hbar^2 k_2^2 \cos^2 k_2 L_2 + \hbar^2 k_3^2 \cos^2 k_3 L_3 + m_0^2 c^2 \right)^{1/2}. \tag{3.8}$$

For simplicity, we shall consider the two – dimensional case for particles with initial mass $m_0 \neq 0$ and $m_0 = 0$.

In the context of C.M. model, the parameter $\tau_1 E_1/\hbar$ plays an important role and we distinguish the following three cases:

$$\tau_1 E_1/\hbar < 1, \qquad \tau_1 E_1/\hbar = 1, \qquad \tau_1 E_1/\hbar > 1. \tag{3.9}$$

For the case $\tau_1 E_1/\hbar = 1$ and $k = 0$, we get

$$\tau_1 = \frac{\hbar}{m_0 c^2} \tag{3.10}$$

which exactly gives the elementary time of particles with rest mass $m_0 \neq 0$.

From the relation

$$\left|\frac{\tau_1 E_1}{\hbar}\right| = 1 \qquad or \qquad \tau_1 = \frac{\hbar}{E_1} \tag{3.11}$$

that corresponds to the threshold, we obtain the Caldirola time τ_1. Moreover, equation (3.11) relates the parameter τ_1 to the relativistic energy, so that for a negative energy we get a negative τ_1. Hence the sign of τ_1 is connected to the particles and the antiparticles and the other cases $\tau_1 E_1/\hbar < 1$ and $\tau_1 E_1/\hbar > 1$ correspond to the stable and unstable eigenstates respectively.

In the context of (S.D.D.) model the formula (3.9) for the two dimensional case takes the form

$$\hbar\omega \cos \tau_2 \omega = \pm c\sqrt{\hbar^2 k^2 \cos^2 Lk + m_0^2 c^2} = \pm c\sqrt{p^2 \cos^2 \frac{pL}{\hbar} + m_0^2 c^2} \tag{3.12}$$

For $\tau_2 = 0$ and $m_0 = 0$ we obtain

$$E = \hbar\omega = \pm cp \cos \frac{Lp}{\hbar} \tag{3.13}$$

which is exactly the new radiation energy. Furthermore for $Lp = \pi\hbar\,(n + 1/2)$, n being an integer, we get $E = 0$ while for $Lp = \pi\hbar n$, we get

$$E = \pm n\pi \frac{\hbar c}{L}$$

which is exactly the energy of the Max Planck lattice. From the relation (3.13) we obtain

$$\omega(k) = \pm ck \cos Lk \tag{3.14}$$

and the group velocity is

$$U_{gr} = \frac{d\omega}{dk} = \pm c\,(\cos Lk - Lk \sin Lk)\,. \tag{3.15}$$

For $Lk = 0$, we get $U_{gr}(0) = \pm c$, while for $Lk = \frac{\pi}{2}$ the group velocity is

$$U_{gr} = \pm\frac{\pi}{2}\, c > \pm c \tag{3.16}$$

which is greater than the light velocity. More details about the (S.D.D.) model [21, 22] and its application in physics can be found in ref [25, 26].

Another interesting example, in which the group velocity is obtained by the (C.M.) model, results for the two models appear to confirm Santilli's hypothesis ref [27], that the maximal speed of the propagation of causal signals is not an absolute constant, but depends on the local physical conditions. It takes the value in vacuum and longer (or smaller) than c in the interior of hadronic (or nuclear) matter. This hypothesis was subsequently investigated by De – Sabata and Gasperini ref.[28], who found the value 75c for the speed of the propagation of causal signals, in the interior of a hadron.

In the following, we will mention the fact that there exists an interaction between particles and space – time in the Lie – admissible formulation, before to apply the (S.D.D.) model for the Wheeler – De-Witt equation. Jannussis ref [29] and Nisioka ref [30], by using the non canonical commutation relations in the Lie – isotopic formulation showed that hadronic mechanics has, in general a non – canonical characteristic time and space. After defining the four momentum in the Lie – isotopic case for the operators q_ν and p_ν, Jannussis obtain the expression for the lifting elements T

$$T = 1 + \frac{q_\nu^{(0)}}{q_\nu}, \qquad T(q_\nu) \neq T(-q_\nu), \qquad q_\nu \geq q_\nu^{(0)} \tag{3.17}$$

The element T is the only one which leaves the usual canonical commutation of q_ν and p_ν invariant. If q_ν is large enough, $T(q_\nu)$ approaches unity. According to ref [27], which was derived via the Lie – admissible formulation and leads to a generalization of the Minkowski space of the proposed models under strong internal forces of the closed non – selfadjoint type.Ref [27] dealt with the question:

"*Can strong interactions accelerate particles beyond the speed of light?*"

According to ref [25, 27] the answer is positive. The relation (3.17) admits general interpretation since it destroys the symmetry of space and time. The destruction of the symmetry of time ($T(t) \neq T(-t)$) is studied by physicists who are particularly concerned with the problem of strong interactions. The element T is also important because its interpretation is compatible with the interpretation of the quantum vacuum by Wheeler ref[31].

In fact, considering the one dimensional case we obtain

$$T = 1 + \frac{L_p}{q}, \qquad q \geq L_p \tag{3.18}$$

so that, for $q \approx 10^{-30} cm$ and L being the Planck length T is of the order of 10^{-3} while for $q = L$, T is doublet. It can be easily seen that for values close to the Planck length L we manifest a change to the topology of space – time in the vacuum state.

Finally, the relation (3.18) considering the fact that there exists a kind of interaction between particles and space – time, leads to some important results which are compatible with experimental ones.

Indeed for $L_p \approx 10^{-33} cm$, $\tau_p \approx 10^{-43} sec$, $l \approx 10^{-15} cm$ and $t \approx 10^{-24} sec$ (where l and t are the elementary length and time of particles), we obtain a distortion of space – time of the order 10^{-18}. The result coincides exactly with the experimental one which is mentioned by Leucs ref. [32] concerning laser experiments performed for the determination on gravitational waves. In addition, relation (3.18) can be utilized for the interaction between various particles and as a result we will also obtain a space – time distortion. Consequently, formula (3.18), accounts not only for the space – time asymmetry but also for the inhomogeneity of it.

A theory of gravity ought to account for the connection between particles and space – time and also for certain problem, concerning quantum cosmology. Some years ago, Gonzalez – Diaz, within the context of the Lie – admissible formulation and using the (S.D.D.) model, achieved the connection between particles and space – time and obtained the energy of the ground state of the universe ref.[33, 34, 35]

Using the (S.D.D.) model, Gonzalez – Diaz, derived the following expression for the four – momentum of a particle:

$$p_l = \hbar \left(\Gamma^j_{jl} + \Gamma^\kappa_{\kappa l} \right) \tag{3.19}$$

where Γ^j_{jl} is the Riemannian affine connection. This relation is of great importance, and according to ref [33] this is the so – called heuristic particle – geometry principle and state that the four – momentum of a particle at a given position is proportional to the Riemannian affine connection at that position. Thus, momentum is directly related to space – time structure.

According to ref.[33], reality ought to be regarded as being dual in a deeper sense: matter and gravitation field should now be the two alternative ways to deal with it.

Also Gonzalez – Diaz in the paper ref.[34, 35], using the concepts of the (S.D.D.) model obtained a new modified Lie – admissible Wheeler – De Witt equation and calculated the energy of the ground state of the universe. Since we give a Lie – admissible structure to the universe, we are forced to consider it as being an open system which is created by some kind of physical reality. Using a minisuperspace approximation consisting in a homogeneous isotopic universe with no matter fields and zero cosmological constant, we finally obtain the eigenvalues

$$E_n \Psi(\alpha) = \pi M^* (n + 1/2) \Psi(\alpha) \qquad n = 0, 1, 2. \tag{3.20}$$

The interpretation of the above equation is that the considered universe has a non – zero energy whose values coincide exactly with the corresponding ones of an harmonic oscillator with Planck mass M^*; it is an open system with intervals with some sort of "exterior" world and it is created by a kind of physical reality.

4 Conclusion

According to the commutation relations (1.2), (1.3) for the case of free Dirac particles and the two forms (2.6), (2.12) of the eigenvalues with $\lambda = \hbar B = \frac{e\hbar H}{c}$, we see that the constant magnetic field H is multiplied by the integer number l for the two cases $\theta = 0$ and $\theta \neq 0$. Also, according to [17, 18] there exists a cut – off of the momentum $\sqrt{l\lambda}$ and a maximum number $l = l_{max}$, which corresponds to a very strong magnetic field on the Max Planck scale. In other words we see that, beginning with a constant magnetic field H by using the above results we obtain a new very strong magnetic field but not infinite. The mechanism that we have used, leads to the existence of stars with very strong magnetic fields, and such stars there exist in the universe.

A few years ago Jannussis *et al* [21, 22] by using the (C.M.) and (S.D.D.) models in the Dirac theory have studied the deformed problem of electrons in a uniform magnetic field and obtain a gauge – dependent electron energy spectrum. The emerging models represent an electron under a uniform magnetic field with non – linear interaction, a spacetime field with non – linear interaction and a space time cut – off, which are relevant to the quantum gravity and other fields.

As Gonsalez - Diaz proved, using the (S.D.D.) model, the four – momentum of particles is now connected with the space – time structure directly, obtained the vacuum energy of universe which is proportional to the Planck mass M^*. Universe is no more considered to be closed and it is considered to be open having a non – zero vacuum energy. Also Gonzalez - Diaz [36, 37] has calculated the curvature tensor which leads to high – derivative terms and form a renormalizable quantum theory of gravitation.

After the generalization of the Wheeler – De - Witt of equation [36, 37] in the Lie – admissible formulation [33], we obtain new results which we have mentioned above. In our opinion, our results are more general than those referred in a paper by Hawking [38]. According to Hawking [38] the effects of little close universe on ordinary particle physics may be small apart, possible, for scalar particles. Nevertheless, it raises an important matter of principle because there is no way in which Hawking could measure the quantum state of closed universe that branch off our nearly flat region, one has to sum over all possible states for such universes. This means that the part of the final state that he can measure will appear to be in a mixed quantum state, rather than in a pure state. But as it is known [39, 40] the study of a mixed quantum system is done with the help of a new density matrix. The theory of this new matrix consists a special case of the Lie – admissible formulation by Jannussis *et al* [41, 42]. As a final conclusion, we see from the till now known results, that the quantum gravity theory and the structure of the universe are well described by the Lie – admissible formulation.

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