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Deformed damped harmonic oscillator

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Abstract

In the present paper we study the deformed harmonic oscillator in the presence of friction. We use the following time dependent Hamiltonian:

$$\mathcal{H} = e^{2\gamma t}\frac{\alpha}{2m}\left(\hat{p}_1 + \frac{\lambda}{2h}\hat{q}_2\right)^2 + e^{-2\gamma t}\frac{\beta m\omega^2}{2}\left(\hat{q}_1 - \frac{\theta}{2\hbar}\hat{p}_2\right)^2$$

where λ, θ are real parameters.

Hamiltonian of this type have been used to study dissipation in quantum mechanics. This is the usual one dimensional Caldirola - Kanai Hamiltonian in the phase space $\hat{P}_1 = \hat{p}_1 + \frac{\lambda}{2h}\hat{q}_2$ and $\hat{Q}_1 = \hat{q}_1 - \frac{\theta}{2\hbar}\hat{p}_2$, where we have insert an extra second dimension $\hat{q}_2$, $\hat{p}_2$. The operators satisfy the commutators of the non commutative geometry, namely

$$[\hat{P}_1, \hat{p}_2] = i\lambda/2 \qquad [\hat{Q}_1, \hat{q}_2] = i\theta/2 \qquad [\hat{q}_2, \hat{p}_2] = i\hbar \qquad [\hat{Q}_1, \hat{P}_1] = i\hbar\mu$$

where $\mu = 1 + \lambda\theta/(4\hbar)$. The second commutator implies the new Heisenberg relation $\Delta Q_1 \Delta q_2 \sim \theta$.

We find the exact propagator of the system. We find as well the time evolution of the coordinates and momenta operators. From the resulting formulas it becomes clear that the system is a new oscillator with frequency $\Omega = \sqrt{ab\,\omega^2\mu^2 - \gamma^2}$.

Finally, we investigate the thermodynamic properties of the system in Boltzmann statistics. We find the statistical density matrix and the partition function $Z(\beta)$.

$$Z(\beta) = \frac{S}{\theta}\frac{2\mu}{\sqrt{1 - \cos(\gamma\hbar\beta)\cosh(\Omega\hbar\beta) - (\gamma/\Omega)\sin(\gamma\hbar\beta)\sinh(\Omega\hbar\beta)}}$$

The term S is a small area and the factor S/θ comes from the spatial non - commutativity.

The specific heat of the system exhibits, for weak friction, an infinite number of singularities (Fig.1) and for strong friction some zeros (Fig.2).

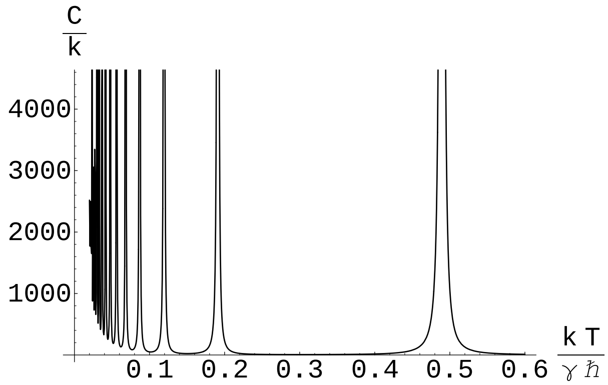

Figure 1: The specific heat for weak friction $\gamma = \frac{1}{2}\sqrt{ab}\omega\mu$

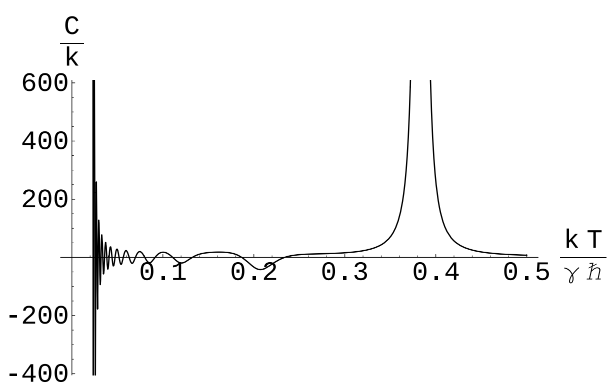

Figure 2: The specific heat for strong friction $\gamma = 2\sqrt{ab}\omega\mu$