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Harmonic Oscillator in non commuting two dimensional space

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Abstract

In the present paper we study the two dimensional Harmonic Oscillator in a constant magnetic field in non commuting space. We use the following Hamiltonian

$$\mathcal{H} = \frac{1}{2m}\left(\hat{p}_1^2 + \hat{p}_2^2\right) + \frac{1}{2}m\omega_1^2\hat{q}_1^2 + \frac{1}{2}m\omega_2^2\hat{q}_2^2$$

with commutation relations

$$[\hat{q}_1, \hat{q}_2] = i\theta, \qquad [\hat{p}_1, \hat{p}_2] = i\lambda \qquad \textit{and} \qquad [\hat{q}_j, \hat{p}_k] = i\hbar\delta_{jk}$$

where λ, θ are real positive parameters. The parameter λ expresses the presence of the magnetic field.

We first prove that the system is equivelant with a two dimensional system where the operators of the momentum and coordinate of the second dimension satisfies a deformed commutation relation.

$$[\hat{Q}_1, \hat{Q}_2] = 0, \quad [\hat{P}_1, \hat{P}_2] = 0 \qquad \textit{and} \qquad [\hat{Q}_1, \hat{P}_1] = i\hbar, \quad [\hat{Q}_2, \hat{P}_2] = i\hbar\mu$$

The deformed parameter $\mu = 1 - \frac{\lambda\theta}{\hbar^2}$ depends on λ and θ and is independent of the Hamiltonian. The two commutative observables of the system are the $\tau_1 = q_1$ and $\tau_2 = q_2 - (\theta/\hbar)p_1$.

Then we write the time evolution operator in a appropriate normal ordered form, so that we can calculate the exact propagator in a straitforword manner. We prove that the unkown fucntions can be found with the help of the solutions of the equivelant classical problem. The method can be applied easily in the case where the frequencies ω_1, ω_2 or the mass m are time dependent. We find as well the time evolution of the coordinates and momenta operators.

We investigate the thermodynamic properties of the system in Boltzmann statistics. We find the statistical density matrix and the partition function which is equivalent of a two dimensional harmonic oscillator with two deformed frequencies $\hat{\omega}_1$ and $\hat{\omega}_2$.

$$\hat{\omega}_{1,2} = \frac{1}{2}\sqrt{(\omega_1 + \omega_2)^2 + (-\omega_1\omega_2\theta + \lambda)^2} \ \pm\ \frac{1}{2}\sqrt{(\omega_1 - \omega_2)^2 + (\omega_1\omega_2\theta + \lambda)^2}$$

Finally we study the case where the phase space of the second dimension becomes classical, that is $\mu = 0 \quad \Longrightarrow \quad [\hat{Q}_2, \hat{P}_2] = 0$. In this case we have one more commutative observable namely $\pi_2 = p_2 + (\lambda/\hbar)q_1$. The final formulas depend now on the following frequencies.

$$\hat{\omega}_1 = \frac{1}{\lambda}\sqrt{(\lambda^2 + \omega_2^2)(\lambda^2 + \omega_1^2)} \quad \textit{and} \quad \hat{\omega}_2 = 0$$

In the case of a free system we find $\hat{\omega}_1 = \lambda$.