

Deformed Harmonic Oscillator for non - Hermitian operator and the Behavior of PT - and CPT - Symmetries

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Abstract

In the present paper we study the deformed harmonic oscillator for the non - Hermitian operator

$$\mathcal{H} = \frac{\alpha}{2m} \left(\hat{p}_1 + \frac{\lambda}{2\hbar} \hat{q}_2 \right)^2 + \frac{\beta m \omega^2}{2} \left(\hat{q}_1 - \frac{\theta}{2\hbar} \hat{p}_2 \right)^2$$

where λ, θ are real positive parameters, since the parameters α, β, m are for the general case complex.

For the case $\alpha = 1, \beta = 1$ and mass m real, we find the eigenfunctions and eigenvalues of energy, the coherent states, the time evolution of the operators \hat{q}_j, \hat{p}_j in the Heisenberg picture and the uncertainty relations. In this case the operator \mathcal{H} is Hermitian and PT - symmetric. Also for the case m complex $\alpha = 1, \beta = 1$, the operator \mathcal{H} is non - Hermitian and no more PT - symmetric, but CPT - symmetric with real discrete positive spectrum and the CPT - symmetry is preserved. In the general case α, β, m complex, for the non-Hermitian operator \mathcal{H} , we obtain complex spectrum and for the special values of the complex parameters α, β the spectrum is real discrete and positive and the CPT - symmetry is preserved. The general problem of deformed oscillator for non hermitian operators can be applied to the Solid State Physics.