The ordering of the operators in Quantum Mechanics

Ph. D. Thesis

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Abstract

In this thesis we study the applications of the theory of the ordering of the operators in quantum mechanics.

With the help of some correspondences between ordered operators and the usual complex functions, the quantum mechanical relations become similar with those of classical mechanics. The system is now described directly in the phase space by a distribution function and the physical observables are represented by the usual complex functions. The symmetric or Weyl correspondence has been studied extensively since the corresponding function of the density matrix operator, coincides with the known Wigner distribution function, We study this Wigner formulation of non relativistic quantum theory and we extend this in the relativistic one. We define the relativistic Wigner operator and we find its eigenfunctions and eigenvalues. We prove that the eigenfunctions are matrices whose elements are Fourier transforms of the product of two Dirac spinors, while the eigenvalues are deferences of the eigenvalues of two Dirac equations.

On the other hand the ordering of the operators simplify the calculation of the action of the operators on the various functions. We write the time evolution operator in an exponential form and we expand this operator in an appropriate form so that its action on the function, follows in a simple and straightforward manner. The resulting function is the propagator of the system which plays a central role in the Feynman formulation of quantum mechanics, which is also developed. For Hamiltonians, time dependent or time independent, which are polynomials of second degree in the canonical conjugate variables, the ordering can be achieved with the method of parametric differentiation. We also evaluate the time evolution of the operators p and q which constitute the independent integrals of motion. Finally we extend the method to the case of relativistic mechanics where we evaluate the propagator of the electromagnetic field.