Abstract

Let \( c_k(b, \nu, a) \) be the \( k \)th positive zero of the function \( bC_\nu(x) + xC'_\nu(x) \), where \( C_\nu(x) = \cos \alpha J_\nu(x) - \sin \alpha Y_\nu(x) \) is the general cylinder function and \( 0 \leq a < \pi \). We prove some results on convexity and concavity of \( c_k(b, \nu, a) \) with respect to the variable \( b \) for \( \nu > 0 \). In particular, we establish lower and upper bounds for \( c_1(b, \nu, 0) \). As a consequence we obtain lower and upper bounds for \( c_1(0, \nu, 0) \equiv j'_{\nu,1} \), the first positive zero of the \( J'_\nu(x) \), which are sharper than previously known ones.