Abstract

Let \( \{P_n(x)\}_{n=0}^{\infty} \) be a system of polynomials satisfying the recurrence relation

\[
P_{-1}(x) = 0, \quad P_0(x) = 1, \quad P_{n+1}(x) + h_n P_{n-1}(x) + c_n P_n(x) = x P_n(x),
\]

where \( h_n, \, c_n \) are real sequences and \( h_n > 0, \, n = 0, 1, 2, \ldots \). The co-recursive polynomials \( \{P^*_n(x)\}_{n=0}^{\infty} \) satisfy the same recurrence relation except for \( n = 1 \), where \( P^*_1(x) = \gamma x - c_0 - \beta, \, \gamma \neq 0 \). It is well known that the problem of determining the zeros of \( P_n(x) \) is equivalent to the problem of determining the eigenvalues of a generalized eigenvalue problem \( T f = \lambda A f \), where \( T \) and \( A \) are symmetric matrices.

In this paper the problem of determining the zeros of the co-recursive polynomials is reduced to a perturbation problem of the operators \( T \) and \( A \) perturbed by perturbations of rank one. A function \( \phi(\lambda) = \phi(\lambda, \lambda_1, \lambda_2, \ldots, \lambda_k) \) is found, \( k = 1, 2, \ldots, n \), whose zeros are the zeros of \( P^*_n(x) \), and \( \lambda_k \) are the zeros of the polynomial \( P_n(x) \) of degree \( n \), for \( \gamma \neq 0 \). This function unifies many results concerning interlacing between the zeros of \( P_n(x) \) and \( P^*_n(x) \) for \( \gamma \neq 0 \). Moreover we obtain from this function similar results in the unstudied case \( \gamma = 0 \).